BACKANALYSIS OF SHEETPILE WALL TEST KARLSRUHE (1993) APPLYING INVERSE ANALYSIS

K.J. Bakker Delft University of Technology , Delft, The Netherlands
Public Works Department; "Rijkswaterstaat", Utrecht, The Netherlands

ABSTRACT: The Finite Element modelling of sheetpile walls has been evaluated in the light of the measurements of the 1993 sheet-pile wall in Karlsruhe. The method applied is a simplified version of the Maximum Likelihood approach, as used by Ledesma (1989), applying the Inverse analysis equations and FEM analysis subsequently. A reasonable fit for stresses and displacements was found, including the force deformation curve for the strut, which was not a part of the fit. The soil stiffness based on the laboratory test result seemed to have under estimated the in situ stiffness, as observed, largely.

INTRODUCTION

In 1993 at the test-site Hochstetten near Karlsruhe, a sheet-pile wall test was performed. The test was organised by the University of Karlsruhe in co-operation with the Dutch Centre for Research and Codes; CUR (Gouda). In advance a prediction contest was held. The test itself, and the prediction results where published by von Wolffensdorfer (1997). The back-analyses

Figure Fout! Onbekende schakeloptie-instructie. **The**

included used by him focused among other things on the material model used; hypoplastic model. The best fit for the parameters was found, as far as could be observed, based on trial and error.

Here in this paper, Bayesian analysis (Ledesma 1989), is used to fit parameters for a Finite Element analysis with PLAXIS, see Vermeer (1995). For the material model the hard-soil model was chosen; a stress dependent stiffness, and hyperbolic stress-strain relation between strain and deviatoric stress in the elastic range, a distinction between primary loading and unloading/reloading, and failure according to the Mohr-Coulomb theory.

TEST EXECUTION AND MEASUREMENTS

The test, was performed in sandy soil, and was heavily instrumented. The test was carried out from the end of may to the begin of June 1993. the final loading was carried out on the 8th of June.

As the ground water level was 5.5 m below soil surface, it has to be considered that the sand showed some apparent cohesion due to suction. The test itself was performed executing the following stages of construction, see table 1:

Table Fout! Onbekende schakeloptie-instructie.

Stage	Stage	Description
0		Initial conditions
1		Excavation up to 1.00 m.
2		excavation to -1.75 m.
3		Installation of the struts and pretension to 4.29 kN/m.
4		Excavation to -3.00 m
5	**I**	Excavation to -4.00 m.
6	**II**	Excavation to -5.00 m
7	**III**	Surface load (in order to reduce the effect of the apparent cohesion).
8	**IV**	Release the strut length up to

Preliminary to the test, after that the instrumented sheet piles where placed but before excavation, horizontal soil stresses where measured, see Fig 2. According to von Wolfferdorff; *"the initial horizontal stress as observed are quite in disagreement with 'as expected' distributions, but nevertheless have to be considered accurate as the measurement was observed four times independently, and showing a coherent view"*. One of the critical things to be predicted was the deformation, (of the strut) at soil failure. As there might be a dispute whether this deformation is well defined, here a comparison between strutforce and deformation will be made. In Fig 6 a comparison between back-analysis and measurement is given. As one can observe, only minor deformations of the wall lead to diminishing values of the strutforce (and apparently to the soil loading of the wall).

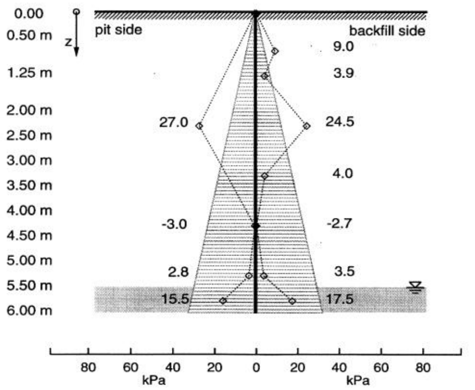

Figure Fout! Onbekende

PREDICTION CONTEST

Among other predictors the Civil Engineering Division of the “Rijkswaterstaat” made two predictions. One with an engineering model based on a Subgrade Reaction Model, the other one with a Finite Element Model; PLAXIS. The prediction was discussed in a paper by Bakker & Beem in the former conference in Manchester (1994).

An elaborate description of the predictions and of the test results, was given by von Wolfferdorff (1994), and presented at a Workshop held in Delft at Delft University, October 6 and 7, 1994. One of the characteristic results presented; here repeated in Fig 4 and 5, is a comparison between all the predictions, and the FEM predictions; PLAXIS, for stage III of the test, (When the pit is excavated, and after placing the “water surcharge” load), showing the bandwidth in predictions. Looking at these pictures one is tempted to derive an estimate for the standard deviation of models, estimating this from the predicted bending moments by; $\sigma \approx (\frac{\max(M)-\min(M)}{4})$. It must be considered however that although all the predictors where based on the same set of parameters, the transformation between, bare geotechnical survey data, and model parameters could be and will have been diverse. Therefor a large part of the standard deviation found would thus have to be attributed to the parameters, and not to the model itself.

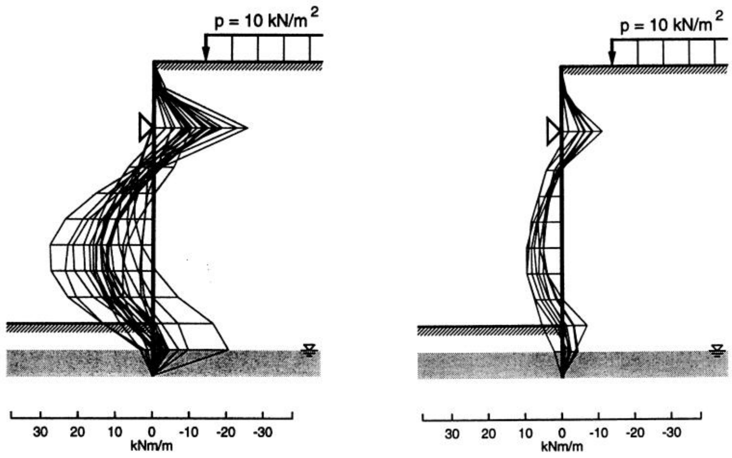

Figure Fout! Onbekende schakeloptie-instructie. **Prediction results stage III, after excavation and surcharge load; left; 4a), all predictions FEM. right; 4b) PLAXIS results**

THEORY FOR THE BACK ANALYSIS

In order to perform a postdiction for Karlsruhe sheet piling test, “Inverse analysis” Ledesma (1989), Nova (1995), was applied.

In this theory, to begin with, an explicit model relating parameters; **x**, and postdiction results; $\mathbf{f}^c$ (where ^{c}, stands for ‘calculation’), has to be available;

$$\mathbf{f}^c = \mathbf{M}(\mathbf{x}) \qquad \text{(Fout! Onbekende schakeloptie-instructie.)}$$

The results of which (the post diction), might be evaluated in relation to measurements; $\mathbf{f}^t$, (where ^{t}, stands for test). Both $\mathbf{f}^c$ and $\mathbf{f}^t$ are assumed to be vectors here, with a length $\boldsymbol{n}$; the number of measurements taken in consideration. Here only a limited number of measurements will be used to fit the parameters; e.g. a maximum bending moment, a strut force and/or a maximum deformation, for a number of successive steps in the excavation, i.e. the engineering parameters being used in the evaluation against construction criteria.

The measurements being taken in consideration and the calculation results of the model might be ordered in vectors according to;

$$f_i^t = (f_1^c, f_2^c, ..., f_n^c)^T$$ (Fout! Onbekende schakeloptie-instructie.)

and

$$f_i^c = (f_1^c, f_2^c, ..., f_n^c)^T$$ (Fout! Onbekende schakeloptie-instructie.)

After Ledesma, (1989), it is assumed that the probability distributions of the prior information of the parameters and the measurements are multivariate Gaussian;

$$P(\mathbf{x}) = \left|\mathbf{C_x^0}\right|^{-1/2} (2\pi)^{-m/2} \exp\left[-\frac{1}{2}\left(\mathbf{x} - \langle\mathbf{x}\rangle\right)^{\mathbf{T}} \left(\mathbf{C_x^0}\right)^{\mathbf{-1}} \left(\mathbf{x} - \langle\mathbf{x}\rangle\right) \right]$$ (Fout! Onbekende schakeloptie-instructie.)

and

$$P(\mathbf{f^c}) = \left|\mathbf{C_f}\right|^{-1/2} (2\pi)^{-n/2} \exp\left[-\frac{1}{2}\left(\mathbf{f}^t - \mathbf{f^c}\right)^{\mathbf{T}} \left(\mathbf{C_f}\right)^{\mathbf{-1}} \left(\mathbf{f}^t - \mathbf{f^c}\right) \right]$$ (Fout! Onbekende schakeloptie-instructie.)

Where;

- $\mathbf{C_x^0}$ is the covariance matrix, based on the available 'a priori' information.
- $\mathbf{C_f}$ measurements covariance matrix
- $\langle\mathbf{x}\rangle$ " a priori" estimated value's of parameters, e.g. the mean value's
- $\mathbf{f^t}$ the measured variable values
- m is the number of parameters evaluated
- n is the number of measurements
- $()^T$ is used to indicate a transpose

If the measurements and the 'a priori' estimates for the parameters are independent, the likelihood of a combination of a priori parameters and measurements is assumed according to;

$$L(\mathbf{x}) = k\, P(\mathbf{x})\, P\left(\mathbf{f^c}\right)$$ (Fout! Onbekende schakeloptie-instructie.)

where k is an arbitrary constant.

The most likely combination of parameters to fit the measurements can be found, solving the minimum of the natural logarithm, which yields the same optimum, as the latter function is monotone. Therefor an additional function S is postulated to be minimised;

$$S = -\ln L(\mathbf{x})$$ (Fout! Onbekende schakeloptie-instructie.)

Which written out yields:

$$S = \left(\mathbf{f^t} - M(\mathbf{x})\right)^T \mathbf{C_f^{-1}} \left(\mathbf{f^t} - M(\mathbf{x})\right) + \left(\mathbf{x} - \langle \mathbf{x} \rangle\right)^{\mathbf{T}} \left(\mathbf{C_x^0}\right)^{-1} \left(\mathbf{x} - \langle \mathbf{x} \rangle\right) + \\ + \tfrac{1}{2} \ln \left|\mathbf{C_f}\right| + \tfrac{1}{2} \ln \left|\mathbf{C_x^0}\right| + \tfrac{n}{2} \ln(2\pi) + \tfrac{m}{2} \ln(2\pi) - \ln k$$

(Fout! Onbekende schakeloptie-instructie.**)**

If the error structure of the measurements and parameters is considered to be fixed, only the first to terms of the equation have to be considered in the minimisation process, the other terms being constants;

$$S^* = \left(\mathbf{f^t} - M(\mathbf{x})\right)^T \mathbf{C_f^{-1}} \left(\mathbf{f^t} - M(\mathbf{x})\right) + \left(\mathbf{x} - \langle \mathbf{x} \rangle\right)^{\mathbf{T}} \left(\mathbf{C_x^0}\right)^{-1} \left(\mathbf{x} - \langle \mathbf{x} \rangle\right)$$

(Fout! Onbekende schakeloptie-instructie.**)**

It is assumed that here that the results of the numerical analysis; $\mathbf{f^c}$ may be expanded using a linear Taylor's expansion according to;

$$\mathbf{f^c} = \mathbf{f_o^c} + \frac{\partial \mathbf{f^c}}{\partial \mathbf{x}} \Delta \mathbf{x} = \mathbf{f_o^c} + \mathbf{A} \Delta \mathbf{x}$$

(Fout! Onbekende schakeloptie-instructie.**)**

Combination of equations 9 and 10 lead to;

$$S^* = \left(\mathbf{f^t} - \mathbf{f_0^c} - \mathbf{A}\Delta\mathbf{x}\right)^T \mathbf{C_f^{-1}} \left(\mathbf{f^t} - \mathbf{f_0^c} - \mathbf{A}\Delta\mathbf{x}\right) + \left(\mathbf{x} - \langle \mathbf{x} \rangle\right)^{\mathbf{T}} \left(\mathbf{C_x^0}\right)^{-1} \left(\mathbf{x} - \langle \mathbf{x} \rangle\right)$$

(Fout! Onbekende schakeloptie-instructie.**)**

Because we intend to improve the solution with respect to trial values of the parameters **x, (**related to the trial values of $\mathbf{f^c}$); $\mathbf{f_0^c}$. If we use the notation $\Delta\mathbf{f} = \mathbf{f^t} - \mathbf{f_o^c}$, equation 11, yields;

$$S^* = \left(\Delta\mathbf{f} - \mathbf{A}\left(x - x^{tr}\right)\right)^T \mathbf{C_f^{-1}} \left(\Delta\mathbf{f} - \mathbf{A}\left(x - x^{tr}\right)\right) + \left(\mathbf{x} - \langle \mathbf{x} \rangle\right)^{\mathbf{T}} \left(\mathbf{C_x^0}\right)^{-1} \left(\mathbf{x} - \langle \mathbf{x} \rangle\right)$$

(Fout! Onbekende schakeloptie-instructie.**)**

Equation 12 can be minimised, differentiating by **x**;

$$\frac{\partial S^*}{\partial x} = -\mathbf{A^T C_f^{-1}} \Delta\mathbf{f} - \mathbf{A^T C_f^{-1} A}\, \mathbf{x^{tr}} + \mathbf{A^T C_f^{-1} A}\, \mathbf{x} + \mathbf{C_x^{-1} x} - \mathbf{C_x^{-1}} \langle \mathbf{x} \rangle = 0$$

(Fout! Onbekende schakeloptie-instructie.**)**

Rearranging the equation in a dependant part with the unknown parameters **x** on the left side, and the a priori information; trial values and a priori values of the unknowns on the right hand, yields;

$$\left(\mathbf{A^T C_f^{-1} A} + \mathbf{C_x^{-1}}\right)\mathbf{x} = \mathbf{A^T C_f^{-1}} \left(\Delta\mathbf{f} + \mathbf{A}\, \mathbf{x^{tr}}\right) + \mathbf{C_x^{-1}} \langle \mathbf{x} \rangle$$

(Fout! Onbekende schakeloptie-instructie.**)**

Equation 14 is the general form for the Maximum Likelihood formulation for back-analysis. If the a priori information is not taken in consideration, the solution simplifies to;

$$\left(\mathbf{A^T C_f^{-1} A}\right)\mathbf{x} = \mathbf{A^T C_f^{-1}}\left(\Delta\mathbf{f} + \mathbf{A\,x^{tr}}\right)$$ **(**Fout! Onbekende schakeloptie-instructie.**)**

Finally, if the error structure matrix is the identity; = one, the more common form of the least squares formulation is obtained;

$$\left(\mathbf{A^T A}\right)\mathbf{x} = \mathbf{A^T}\left(\Delta\mathbf{f} + \mathbf{A\,x^{tr}}\right)$$ **(**Fout! Onbekende schakeloptie-instructie.**)**

FINITE ELEMENT MODELLING

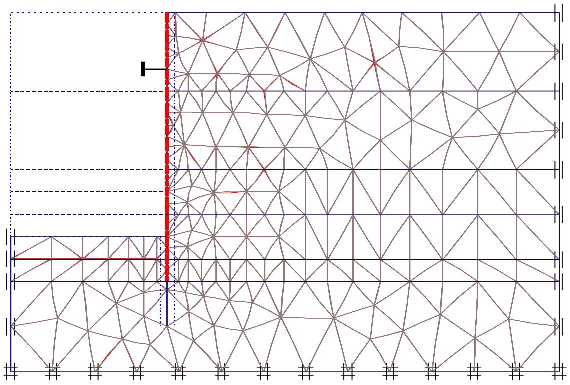

Figure Fout! Onbekende schakeloptie-instructie. **Finite Element model for post diction**

The FEM analyses both for the prediction as well as for the postdiction were performed with PLAXIS, the prediction with version 4.5, and the postdiction with version 7. The test is modelled in plane strain. The mesh for the postdiction is given in Fig. 4. The mesh displayed is the mesh at a certain stage of construction, i.e. the soil elements in the pit are removed yet. In the initial situation a level soil surface is modelled. In order to improve the analysis, all stages of the test are modelled and analysed subsequently.

The shortening of the struts in the final stage of the analysis was performed by removing the strut in a staged construction phase, up to the point in the analysis that the soil yields.

PARAMETERS AND MEASUREMENTS TO EVALUATE

After the test comparing measurements and predictions, discussion focused on 1) the soil stiffness; i.e. for small strains, 2) Apparent cohesion due to suction, 3) initial stresses due to the installation procedure. Here the following considerations where made;

Friction angle; apparently the soil is 'stronger' than anticipated in the predictions. The bending moments and strut-forces are largely over estimated in the predictions Therefor, in the back-analysis to begin with a friction angle at failure (the top value, instead of at $\varphi_m(3\%)$), will be assumed, i.e. $\varphi = 42^{\circ}$.

Apparent cohesion; In the back-analysis by von Wolffensdorfer (1996), it is mentioned that for the top-layer of 1.5 m approximately, a capillary underpressure of approx. 13 kPa is active, leading to an apparent cohesion of C_{uns} = 13 tan(42) = 11.7 kPa.

Elasticity of the soil; In the prediction by Beem & Bakker (1994), the Mohr-Coulomb model with a G_{50} was used. With this approach, the unloading of the soil, with a much stiffer behaviour was disregarded. In the back-analysis, the PLAXIS 'hard-soil' model; with an hyperbolic strain hardening relation acc. to Duncan & Chang, 1970) is applied. The hard soil model, see Vermeer & Brinkgreve (1995), identifies a Initial Young's modulus; E_i and unloading-reloading Young's modulus E_{ur}. As a trial value, the modulus from the Triaxial-test results is $E_{50} \approx 2(1+\upsilon)G_{50}^{ref} \approx 35000$ is used. Subsequently the Cone-penetration results have been looked at, with respect to the emperic relation that $E \approx (3\ to\ 5)\, q_c$. Based on that 5 layers with a different stiffness have been distinguished..

For the unloading reloading modulus, according to the a priori data set for the test, the“Platten-druckversuch”; the load plate test, the stiffness ratio, for unloading reloading is 1.6. As the initial stiffness, E_i, assuming an hyperbolic shape for the hardening curve is twice the value at E_{50}, the Young’s modulus for unloading reloading E_{ur} is assumed to be 1.6*2 = 3.2 times E_{50}. The young’s moduli used are gathered in Table 2

Initial stresses; The earth pressure measurements, in advance of the excavation, see Fig 2 indicate that in the upper zone, 2.0 m an increased horizontal stress is active. Approximately twice the value acc. to Jaky; $K_0 \approx (1-\sin\varphi)$, was observed. Whereas below 3.5⁻ soil surface, the horizontal stresses seem to contradict with plasticity theory, as for active failure; $K_0 \approx \frac{1-\sin\varphi}{1+\sin\varphi} - \frac{c}{\sigma_v}\cos\varphi$.

For a depth of 3.5 m. e.g. with a $\sigma_v \approx 3.5 * 16.5 \approx 58\,kPa$, a soil friction of approximately $\varphi \approx 42^0$ and a cohesion of $c \approx 5kPa$ this would lead to a minimum value of $K_0 \approx 0.14$.

The observed K_0 value of appr. 0.0 suggests a Cohesion of appr. 15 kPa which is considered to be unrealistic. In the postdiction analysis, a value for K_0 for the below 3.5 m of 0.2 is used , whereas the value for the undeeper layers was being considered a free parameter in the optimisation.

Table Fout! Onbekende schakeloptie-instructie. **Soil data used as initial values in the back analysis**

Fout! Bladwijzer niet gedefinieerd.Fout! Bladwijzer niet gedefinieerd. Layer top +. MSL	γ_d kN/m³	γ_n kN.m³	φ [°]	ψ [°]	c kPa	c_{depth} kPa	Ref C_{depth} m	E_{50}^{ref} kPa	E_{ur} kPa	υ [-]
+0.00	16.9	-	42.0	12.0	11.7	-	-	65000	208000	0.3
- 1.25	16.5	-	42.0	12.0	11.7	-2.52	-1.25	65000	208000	0.3
- 3.50	16.5	-	42.0	12.0	11.7	-2.52	-1.25	35000	112000	0.3
- 4.50	16.5	-	42.0	12.0	1	-	-	70000	224000	0.3
- 5.50	16.5	19.0	42.0	12.0	1	-	-	35000	112000	0.3

BACK ANALYSIS

The measurements taken in consideration for the back-analysis, are a subset of the total measurements. This subset of characteristic measurements, such as anchor force, bending moment, and maximum deformation is evaluated for several stages of construction. The measurements are: M2(2.0), which stands for the bending moment in the second stage of excavation, at the height 2.0 m below the top, M6(1.0), M6(3.0), F6, where F stands for the strutforce, U7(0.0); the displacement in stage 7 at the top, U7(3.10), M8(2.0), M8(3.0) and finally F8.

The solution of equation 14 and 15 demands that a covariance matrix for the measurements is established. This would not be necessary for the plain least squares approach acc. to equation 16, which implicitly assumes a standard deviation for the measurements of 1.0.

In order to weigh the importance of the measurements, and to do this in a way not too subjective, here it was assumed that measurements are independent, off diagonal terms are assumed to be zero, whereas a variance; $\sigma_i / \mu_i = 0.1$ is adopted. Above that error's $\left(\Delta f_i\right)$ with respect to bending moments and strutforces are weighed heavier in comparison than deformations, for a factor 5.

OBSERVATIONS OF THE BACK-ANALYSIS

The back analysis was started, with the weighed Least Square approach acc. to equation 15 The method itself was applied by extracting the gradients assembled in the A matrix, from the EEM model only once, consecutively improving the solution iteratively, updating the trial value for a next step using the result of the former LS analysis. The assumption implicitly made here was that the derivatives of the model, for the reach in consideration are not too strongly depending from the position of the model in the solution space. The necessity to do so was that the FEM model could only be handled menu driven so that the extraction of the derivatives for the matrix A could in practice not be made automatically.

This procedure appeared to be reasonably stable, giving convergence in approximately 15 steps. Within this process, equation 14 was solved using Mathcad ©.

One of the final results of this analysis was that the Ratio for the Young's modulus came out nearly four times as high as the value extracted from the triaxial test results, presented in table 2. After the LS result was derived the Maximum Likelihood formulation according to equation 14 was used in order to try to improve the result. It came out very soon that the procedure without updating matrix A, did not yield a good converge. The improved solution is given in table 3, though it has to be mentioned that only, 2 or 3 convergent iterations could be made, depending on the relaxation factor applied; the smaller, the more steps, the less the improvement derived. After that divergence appears. The a priori information seems to yield higher cohesion values.

A comparison of back-analysis results and measurements is given in table 3.

The terminology is, that M stand for bending moment, U for displacements, and F for the strutforce. The parameters derived in the subsequent analyses are listed in table 4,

Table Fout! Onbekende schakeloptie-instructie. **Post diction results**

	k0(1)	**K0(2)**	**φ**	**ψ**	**R(δ)**	**C(1)**	**C(2)**	**R(E)**
Prediction	0.38	0.38	38	5	0.66	5	5	1.0
Least squares	2.85	0.6	42.7	4.7	1	6.35	5.5	4.25
Maximum Likelihood	2.92	0.59	42.5	6.0	1	7.2	5.85	4.1

where

$K_0(1)$ = the K_0 in the upper soil layer until 1.25 m. deep
$K_0(2)$ = the K_0 in the soil layer between 1.25 m. and 3.5 m. deep
C(no) = The cohesion in soil layer (no) (numbering is top downwards)
R() = the Ratio with respect to 1) δ; wall friction and 2) E; Young's modulus, (for all soil layers)
φ and ψ are varied for all soil layers

Finally in Figure 5, The soil pressure, bending moments and displacements are displayed for stage 7 . The displacement anchor-force plot is given in Fig 6. As one can observe for the ultimate values of stresses; bending moments, a reasoble agreement hase been derived whereas the distribution indicates that the soil loading, in situ, is acting on a higher level than in the model. Apparently the stiffness of layer 3 is not conform the assumptions acc. to table 2. The distribution of stiffnesses could therefor be optimized which was not a part of this analysis. With respect to displacements; the actual stiffnes of the strut is appr. 30 % less, yielding the disagreement for the hor. displacements caused by the swing.

Table Fout! Onbekende schakeloptie-instructie. **Comparison Measurements and back analysis**

		Prediction	**Measurement**	**Back-analysis Least squares**	**Back-analysis Maximum Likelihood**
Bending moment stage 2	M2(2.00)	pm	2.26	1.93	1.99
	M6(1.00)	-5.8	-4.41	-5.159	-4.96
Field moment stage 6	M6(3.00)	5.38	2.2	1.778	1.65
Strut force stage 6	F6	23.36	28.64	29.68	28.38
Field displacement stage 6	u6(3.00)	3.51	2.99	2.637	2.49
Head moment stage 7	M7(1.25)	-6.78	-5.06	-6.234	6.03
Field moment stage 7	M7(3.00)	6.72	2.76	2.138	1.99
Strut force stage 7	F7	30.07	33.72	34.86	33.91
Top displ. Stage 7	U7(0.00)	-0.586	5.15	2.86	2.90
Field displ. Stage 7	U7(3.00)	7.19	3.4	3.27	3.14
Ultimate bending moment	M8(2.00)	5.99	4.67	3.919	3.87
Ultimate bending moment	M8(3.00)	12.2	3.41	9.41	9.44
Ultimate strutforce	F8	10.0	4.22	3.035	3.38

CONCLUDING REMARKS

A reasonable fit of the parameters has been derived. Apart from aspects such as the importance of initial stresses, and the underestimated influence of under-pressure in the soil, it appeared that in this case the stiffness based on triaxial cell tests strongly underestimated the observed behaviour. With respect to wall friction; the common used value of 2/3 of the soil friction underestimes practice.

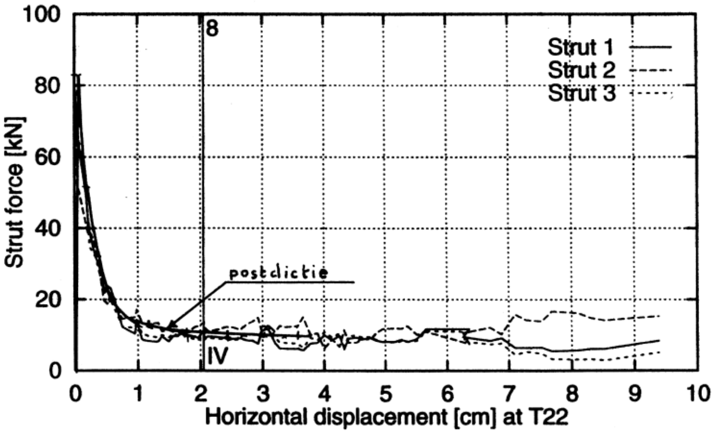

Figure Fout! Onbekende schakeloptie-instructie. **Strut**

It is thought too, that the description of small strain behaviour is largely improved by the Hard soil model. For convergence of the Maximum Likilihood analysis, an update of the gradient matrix A seems to be necessary to derive convergence.

REFERENCES

BAKKER, K.J. & BEEM R.C.A. (1994)
Modelling of the sheet pile wall test in Karlsruhe 1993
Proc. 3the European conf. on Numerical methods in
Geotechnical Engineering; ECONMIG 94, Manchester/UK/7-9-sept.
DUNCAN J.M.., CHANG C.Y. (1970)

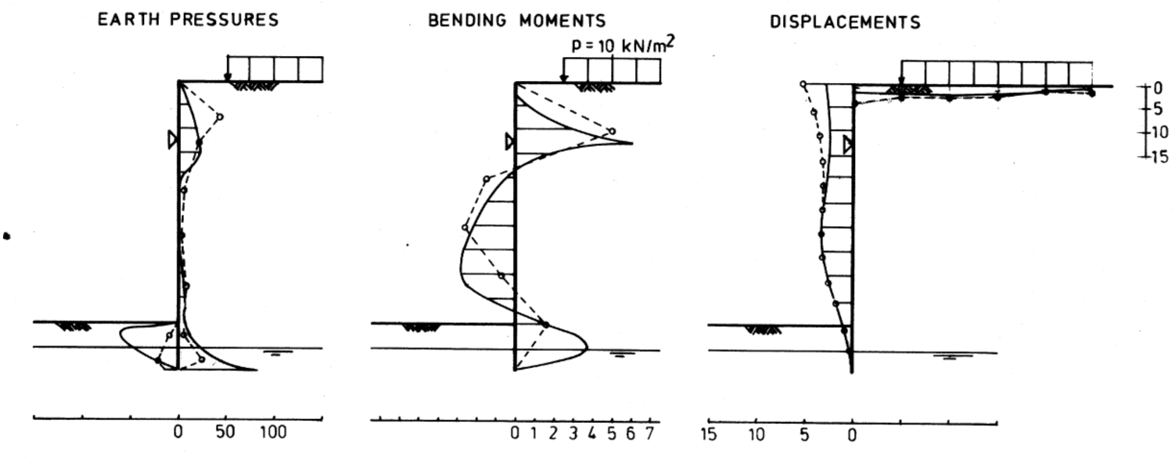

Figure Fout! Onbekende schakeloptie-instructie. **Comparison of postdiction results and measurements**

Nonlinear Analysis of Stress and Strain in Soil,
ASCE J. of the Soil Mech. and Found. Div. Vol 96
NOVA R., BOULON M.,GENS A., (1995)
Numerical modelling and information technology, Theme Report 6,
In: The Interplay between Geotechnical Engineering and Engineering Geology,
Proc. XI European Conf. on Soil Mechanics and foundation Engineering, Copenhagen
LEDESMA, A. GENS, A., ALONSO E.E. (1989)
Probabilistic backanalysis using a maximum likelihood approach
XII ICSMFE, Rio de Janeiro.
VERMEER, P.A. and BRINKGREVE, (1995)
PLAXIS, Finite Element Code for Soil and Rock Analysis
Balkema, Rotterdam.
WOLFFENSDORFER, P.A. von (1994)
Workshop Sheet Pile Test Karlsruhe
Delft University, October 6 and 7, 1994
WOLFFENSDORFER, P.A. von (1995)
Vervormungsprognosen von Stutzkonstruktionen.
Habilitationsschrift, Universitat Fridriciania Karlsruhe