CHAPTER 3

Development of models for structural analysis

3.1 INTRODUCTION

In chapter 2, structural design has been discussed in general terms. What we have seen is that the design process is composed of a creative effort, where feasible alternatives are brought forward, and analytical effort, where the alternatives are optimised. The second aspect of structural design that we have seen is that in the optimisation step a balance is sought between the value of the function being created, and the investment.

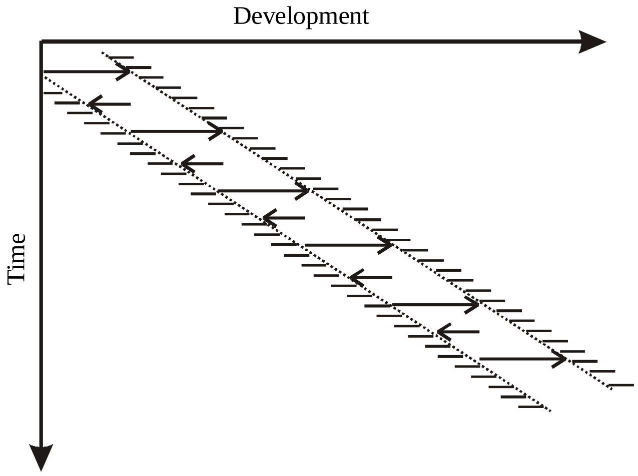

Figure 3.1 Design; two steps forward, creating an alternative, one step backward evaluating its performance.

For structures, where the risk of structural failure, i.e. collapse, always plays an important roll, it has to be remembered that the risk of failure is implicitly included in the cost. This risk might dominate above other value components if not taken into account properly. Therefore it is a common acknowledged fact that structural analysis is an important part of the design. The evaluation of the ultimate limit state is therefore mandatory.

Another important consideration in the optimisation is that if the structure is not able to sustain its structural integrity in a proper way during its lifetime, (although not failing), the functions it has to fulfil might not be guaranteed. The serviceability of the structure is then at risk.

After the creative step taken first, inevitably, a second step follows where the feasible alternatives are put to the test, and optimised with respect to their performance and cost. Metaphorically, one might say that the creation of a design alternative takes us two steps forward, see Fig. 3.1, whereas subsequently the optimisation and evaluation of the alternative takes us back one step.

The two steps forward is strongly related to the human capability to synthesise a solution, based on a thorough knowledge of all the demands and the resources available. This synthesis is related to our intuition to what is feasible. The backward step is much more analytic and relies on deductive processes and understanding of behaviour. Although we have been put back one step, on the whole the design has made progress.

Structural analysis has to be placed in the context of the backward step; putting design alternatives to the test. That is, analysing whether the behaviour of the structure satisfies the relevant criteria.

3.2 STRUCTURAL ANALYSIS

Here the adjective ‘structural’ is used if the analysis is related to a coherent mechanical system. Though a dike or an embankment is a structure according to this definition, here in this study, the description ‘structural’ is mainly used in the context, where a soil body is combined with a structure, not being a soil body, such as a building on top of the soil, or a structure build up of steel or concrete, on top, or in the soil, such as a sheet pile wall. As there is a large difference in the length scale of a structural element and a soil particle, and related to that a large difference in strength and stiffness, soil retaining structures often show a mechanical behaviour which is distinctly different to that of a soil body, such as a dike or an embankment.

Common acknowledged aspects for the evaluation of a structure in its limit states are:

- *Strength*
- *Stiffness*
- *Stability*

These aspects relate to the behaviour of a structure both in the ultimate limit state, and for the serviceability limit state.

In order to test a feasible structure for its limit state conditions, models are used. The time when a structure was built and then verified by a loading test, i.e. a predefined overloading of the prototype, has now passed. Therefore, modern structural analysis relies on models.

Since the introduction of computers in the second half of the 20th century, the development of numerical models has been strongly enhanced. For structural analysis, and also for groundwater-flow, the development of numerical models based on finite element techniques has strongly enhanced the use of these models. The development and tailoring of such for practical purposes is the main topic of this thesis.

3.3 SOURCES FOR MODEL DEVELOPMENT

3.3.1 *Limit state analysis*

Structural models have to be able to describe the limit states for structures, i.e. the ultimate limit state, and if possible be able to describe behaviour to evaluate the serviceability limit state(s). To begin developing structural models a certain amount of knowledge about the modes of failure is needed. A behavioural model that is unable to describe failure modes is a weak model.

Examples of models based on the description of a single mechanism are

- *Bishop's slip circle analysis for the stability of sloping surfaces*
- *Prandtl's theory of bearing capacity*
- *Brinch Hansen's empirical model for bearing capacity*
- *Blum's theory for sheet pile walls*

Knowledge about failure modes directly or indirectly comes from observations. Case histories describing real failures, for example, are an important source. Laboratory tests on samples or on structural parts may also develop insight into the way that a structure might behave. Based on that, mechanical concepts might be used to evaluate the probability of a feasible mode of failure.

Both direct observations as well as mechanical concepts might be a source for model development.

3.3.2 *Development space*

In addition to observations only, which are a primary source for knowledge about limit states, another source for model development would be the evaluation of models, used and applied in practice, in combination with observations related to these structures. If these observations relate to failures they even have a higher value. Back-analysis of case histories might give us another source.

If one would recognise the existence of a mathematical domain, to develop models to describe structural behaviour, such a development space might be spanned on a finite number of parameters. If there would be an insight where in this development space, the models show an inability to describe observed behaviour, further model development could be directed on this part of the development space.

In this context it is important to have a view on models developed in the past and also on the shortcomings of subsequent models. If a hierarchy, a classification of grades of performance of models, was apparent between subsequent models, this might help to distinguish promising further developments.

The parameters being used as a base for a model are decisive for the model space that comes forward from a model. The addition of additional parameters expands the solution space. Contrary to the arbitrary use of parameters, from a physical point of view a specific hierarchy in the use of parameters is also possible. Based on this the development space for model developments, creating a sequential hierarchy between models along the following lines of model extension, might be recognised:

1. static
2. cyclic
3. dynamic

a. discrete
b. continuum

A. elastic
B. elastic plastic
C. elastic visco-plastic
D. visco-elastic visco-plastic

i one dimensional
ii plane strain/plane stress
iii axi-symmetric
iv three dimensional

B.1 Coulomb friction
B.2 Mohr Coulomb strength
B.3 Tresca
B.4 Von Mises
B.5 Cam-Clay model
B.6 Friction hardening model

Δ. one phase model
Φ. uncoupled two phase; (groundwater flow - stress analysis)
Γ *coupled two phase; (consolidation analysis)*
ϑ *three phase*

Each line of extension might be regarded as an axis in a multi dimensional development space. A coordinate in this development space would characterize a model type.

If weighting would be assigned to each axis in the development space, an explicit hierarchy would become apparent between models. As the weighting assigned to an axis is arbitrary, the only hierarchy recognized is that between models, (which have a similar co-ordinate) only differing on one development axis. To develop an opinion to decide for promising development directions, the evaluation of subsequent models, and observed discrepancy of these models with observations is valuable, taking into account the position of a model with respect to its position in the development space.

Within this concept of a development space, development might proceed by making new combinations, where feasible new models move further away from the origin in this development space. Theoretically this process might proceed up to the point that the space is filled up with models, then the process halts until a new development axis is recognised.

If one chooses a careful position for observation of the models, a certain hierarchy between the models may be recognised. If one validates models of subsequent hierarchy, a diminishing error between observations and model should be observed. Although it must be recognised that the absolute error of a model will not necessarily become smaller when a model is refined. A complex model will only give a better prediction if all the parameters are correctly estimated.

The additional parameters in a complex model may introduce additional uncertainty that counteracts the improved accuracy of the model.

3.3.3 *Verification*

An important step to be taken when developing a new model is verification. Verification in practice means making a comparison with previous models. For these previous models, which are assumed to be lower in hierarchy and therefore simpler, it is assumed that there are a sufficient number of commonly accepted solutions.

Verification is a logical test.

The following models for verification are available

- *Analytical solutions, often elastic solutions*
- *Upper bound solutions applying plasticity theory*
- *Lower bound solutions, equilibrium solutions*
- *Hierarchical models of lower order*

For a specific case, one or more of these models may be used to compare with the new model. For an elastic analysis, the new model should provide a close match. If the material model is used to model associated flow plasticity, then it should provide a solution that is between the upper and lower bound solutions. For non-associative material behaviour, the new model should provide a close match with models of lower hierarchy, for the case that new parameters are assigned neutral values. The new model should be able to exhibit the behaviour of the preceding models in the hierarchy.

3.3.4 *Validation*

The final stage in the development of a model is validation. The term ‘validation’ describes the effort to establish insight into the capability of a model to describe prototype behaviour. This means that a different type of comparison is made, not with other models, but with observations. The following observations might be used for this purpose:

- *Empirical relations (implicit observations)*

- *Physical model tests; e.g. 1D lab tests; centrifuge tests*
- *Prototype tests;*
  - *Prediction/evaluation*
  - *Back-analysis*

Empirical relations are judged here as a form of condensed observations and therefore applicable for validation. Laboratory tests might be used for testing the integrity of important components of a model. Essentially in laboratory tests relatively homogeneous processes are tested, for which the model needs to be able to describe observed behaviour.

Finally prototype measurements, and preferably in combination with a set up where the model is used for predictions, make it possible to do real validation. Where a back-analysis gives an impression about the integrity of a model, the combination prediction versus measurement; evaluation gives a view on the practical value of a model; the power to use the model for design purposes.

If a model is used to investigate the variations of the parameters, the comparison of the results with the measurements might give an impression about the model accuracy. Though this variation implicitly includes the uncertainty of the parameters, the band-with in model outcome gives an impression of the stability of the model. If this band-with is large, it means that a small variation, or uncertainty, in a parameter means a large shift in the outcome. If a parameter is difficult to establish, and the model is sensitive for such a parameter, this would mean that the model is unfit for design purposes, though the model might still be useful for back-analyses purposes.

3.4 THE MERIT OF A MODEL

The merit of a model consists of several components. Good agreement with observations in a back-analysis is the first one. If a model is able to predict tests with a high accuracy, such a model becomes useful for design purposes.

Another aspect is the cost of the development of a model. The specification of a model is developed much faster than that the model is formulated, tested for integrity, verified, and validated. Furthermore if the model will be used by a larger group of users, the cost related to making the model accessible to others must be considered. Efforts related to user-friendliness and education; seminars and workshops needed for the transfer of information to prospective users have to be considered. If the model has to be maintained for a long period, cost related to maintenance of the model and also updates involving adjustments and corrections have to be considered. If the model is a numerical model, the source code has to be maintained and kept up with new releases of compilers and operating systems.

If there is a necessity for a model, such as if the proposed structure cannot be designed and realised within a low enough probability of failure without the accurate model (e.g. rocket to the moon, storm surge barrier), the aforementioned cost are relatively small in comparison to the investment in the main project.

Without a necessity for a model or if the investment is not small in comparison to the main project, the economic evaluation such as discussed in chapter 2 has to be taken into account again.

3.4.1 *Finite element analysis*

The development of numerical models; e.g. finite element analysis, based on continuum mechanics, enables the development of a consistent concept for a unified approach for structural models; i.e. one single concept for all types of structures.

One of the main features of the finite element method is that it is general enough to produce failure mechanisms that have not been included as such in the model, but are incorporated on a deeper level, by using a general field of deformations.

The advantages of such an approach are:

- Unification of model developments. The effort being put in model development for different criteria is less. Developments are not primarily aimed at one application or criteria to evaluate.
- Education. The structural analyst only has to know one type of model
- Maintenance of source code

In this thesis a secondary aim is to evaluate the power of the finite element method to cope with different types of engineering problem. One of the crucial questions then is whether the consistency of the finite element method is more reliable than the implicit strength of empirical models. Empirical models implicitly include aspects such as 3D, dynamics, creep, ageing, which have to be included in finite element analysis in an explicit way.

In the cases treated in chapter 5, 6 and 7, the composition of the finite element method to adjust to a number of practical problems is analysed and evaluated.

3.5 DEVELOPMENT STRATEGY

The development strategy followed in this thesis is related to the development of finite element techniques for practical purposes in civil engineering; i.e. soil retaining structures.

The strategy is composed of the following steps, see Fig. 3.2

1. *Knowledge of the mechanisms and modes of failure.*
2. *Knowledge of the hierarchy of existing models for structural analysis.*
3. *Insight into the inaccuracy of existing models.*
4. *Proposing new models, within the framework of finite element techniques.*
5. *Verification of the models, comparing the results with models of lower hierarchy.*
6. *Making predictions with the new model.*
7. *Performing tests.*
8. *Validation of the model eveluating the new model with respect to the tests results.*

3.6 CONCLUDING REMARKS

In this chapter, a strategy to develop models for structural analysis is discussed. The application domain for developments is mainly aimed at soil retaining structures. Structural analysis is discussed in the context of design of structures. Design of structures is considered as being composed of a creative effort producing feasible alternatives, followed by an analytic effort, where the alternatives are being put to the test and optimised.

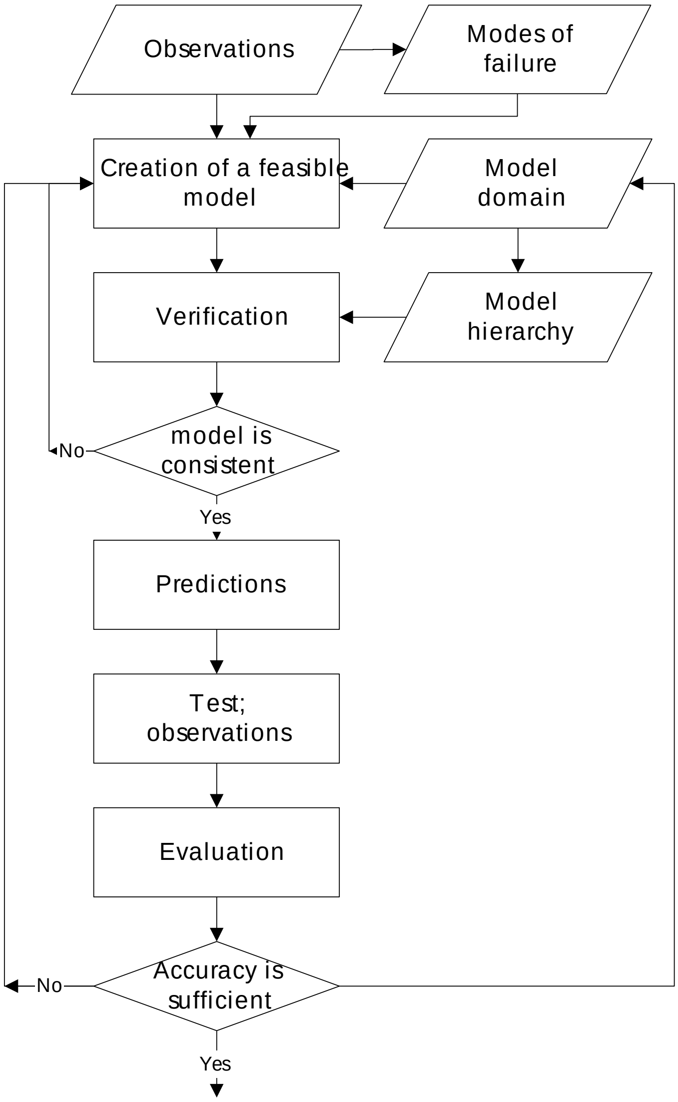

Figure 3.2 Scheme for the development of models for structural analysis

Structural analysis is an important part of the optimisation of a structure. Without knowledge (using models) of the behaviour of a structure, the optimisation merely would become an arbitrary extrapolation, which would be a high-risk process. The sources for the development of models for structural analysis are discussed. The merit of finite element analysis, as a development domain for models for structural analysis was discussed. Finally the development strategy is summarised.